MA131 - Analysis 1
Workbook 1 Assignments

Due in 12th Oct

Assignment 1

1. Solve the inequality \( \frac{1}{x} < x < 1 \) by Case Analysis.

2. Consider the following argument:

\[
\frac{1}{x} < x < 1 \quad \therefore 1 < x^2 \quad \therefore 1 < x.
\]

But \( x < 1 \), therefore there are no solutions. How many mistakes can you find? Comment on this “solution” as though you were a teacher and it was written by one of your students.
Assignment 2

Is the following statement true for all \( x \) and \( y \): “If \( x < y \) then \( x^2 < y^2 \)? What about this statement: “If \( x^2 < y^2 \) then \( x < y \)?

Assignment 3

1. Use induction to prove that if both \( x \) and \( y \) are positive then \( x < y \implies x^n < y^n \).

2. Now try to prove the converse, that if both \( x \) and \( y \) are positive then \( x^n < y^n \implies x < y \).
Assignment 4  Rewrite each of the following expressions without absolute value signs, treating various cases separately where necessary.

1. \( a - |(a - |a|)| \).
2. \( |(|x| - 2)| \).

Assignment 5  Solve the following inequalities:

1. \(|x - 1| + |x - 2| \geq 5\); 2. \(|x - 1| \cdot |x + 1| > 0\).
Assignment 6

1. Put a variety of numbers into the Triangle Inequality and convince yourself that it really works.

2. Write out the triangle inequality when you take $x = a - b$ and $y = b - c$.

3. Prove the Triangle Inequality.

Assignment 7

1. Show, for positive $a$ and $b$, that $\frac{a+b}{2} - \sqrt{ab} = \frac{(\sqrt{a} - \sqrt{b})^2}{2}$.

2. Show that the arithmetic mean is always greater than or equal to the geometric mean. When can they be equal?