Assignment 4
Analysis I
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Due: Monday, 2 November, 3:00pm.

1. Find the limit of each of the sequences defined below

a. $a_n = \sqrt[n]{1^2 + 2^2 + ...n^2}$

b. $a_n = \frac{n+\sin n^2}{n+\cos n}$

c. $a_n = \frac{1-2+3-4+...+(-2n)}{\sqrt{n^2+1}}$

2. Prove or disprove the following statement
"Suppose $(a_n) \to a$. If $a_n > 0$ for all n then $a > 0$."

3. Prove the following theorem:
If $(a_n) \to a$, $(b_n) \to b$ and $a_n \leq b_n$ for all n then $a \leq b$.

4. Let $(a_n) = (n^2)$. Write down the first four terms of the three subsequences (a_{n+1}), (a_{3n-1}) and (a_{2^n}).

5.Prove that every subsequence of a bounded sequence is bounded.

6. Use a calculator to explore the limit of $(2^n + 3^n)^{1/n}$. Now find the limit of the sequence $(x^n + y^n)^{1/n}$ when $0 \leq x \leq y$.

7. Write down the conjectured limit for the power sequence $(a_n = x^n)$ (Warning: you should get four different possible answers depending on the value of x.) Then prove your conjectures.

8. State whether the following sequences tend to zero or infinity. Prove your answers.

(a) $\left(\frac{n^{1000}}{2^n}\right)$ (c) $\left(\frac{n!}{n^{1000}}\right)$

(b) $\left(\frac{(1.001)^n}{n}\right)$ (d) $\left(\frac{(n!)^2}{(2n)!}\right)$

9. Find the limits of the following sequences. Give reasons.

(a) $\left(\frac{n^4 11^n + n^9 9^n}{7^{2n} + 1}\right)$ (c) $\left(\frac{3n^3 + n\cos^2 n}{n^2 + \sin n}\right)$

(b) $\left((4^{10} + 2^n)^{1/n}\right)$ (d) $\left((3n^2 + n)^{1/n}\right)$