Problem 1. Consider the series $1 + 2 + 3 + ... = \sum_{k=1}^{\infty} k$. Give a precise explanation of either (i) or (ii).

(i) $1 + 2 + 3 + ... = \infty$.

(ii) $1 + 2 + 3 + ... = -\frac{1}{12}$.

Problem 2. Find the sum of the series

(a) $\sum_{n=1}^{\infty} \frac{1}{10^n}$.

(b) $\sum_{n=1}^{\infty} \frac{9}{10^n}$.

(c) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$. Hint: the series $\sum \frac{1}{2n-1}$ and $\sum \frac{1}{2n+1}$ may help.

Problem 3. Find $N$ such that

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{N} > 10$$

Give reasons.

Problem 4. Prove the Sum Rule for series.

Problem 5. Prove the Shift Rule for series.

Problem 6. Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ for which $(a_n) \to 0$.

Problem 7. Prove that if $\sum_{n=1}^{\infty} a_n$ converges, then $(a_n) \to 0$.

Problem 8. Use the Comparison Test to decide whether the following series are convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^p}$.

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

(c) $\sum_{n=1}^{\infty} \frac{n + \sin n}{n^2 + \cos n}$.

(d) $\sum_{n=1}^{\infty} \frac{n \sin n}{4^n}$. 