Assignment 7
Analysis I
Daniel Ueltschi

Due: Monday, 23 November, 3:00pm.

Problem 1. Consider the series $1+2+3+\ldots=\sum_{k=1}^{\infty} k$. Give a precise explanation of either (i) or (ii).

(i) $1+2+3+\ldots=\infty$.

(ii) $1+2+3+\ldots=-\frac{1}{12}$.

Problem 2. Find the sum of the series

(a) $\sum_{n=1}^{\infty} \frac{1}{10^n}$.

(b) $\sum_{n=1}^{\infty} \frac{9}{10^n}$.

(c) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$. Hint: the series $\sum \frac{1}{2n-1}$ and $\sum \frac{1}{2n+1}$ may help.

Problem 3. Find N such that

$$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N}>10$$

Give reasons.

Problem 4. Prove the Sum Rule for series.

Problem 5. Prove the Shift Rule for series.

Problem 6. Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ for which $(a_n) \to 0$.

Problem 7. Prove that if $\sum_{n=1}^{\infty} a_n$ converges, then $(a_n) \to 0$.

Problem 8. Use the Comparison Test to decide whether the following series are convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^n}$.

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

(c) $\sum_{n=1}^{\infty} \frac{n+\sin n}{n^2+\cos n}$.

(d) $d.$ $\sum_{n=1}^{\infty} \frac{n \sin n}{4^n}$.