Assignment 9
Analysis I
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Due: Monday 7 December, 3:00pm.

Problem 1. Let $s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. Use $|s - s_n| \leq \frac{1}{n}$ to find a value of N so that

$$|\sum_{n=1}^{N} \frac{(-1)^{n+1}}{n} - s| \leq 10^{-6}.$$

Problem 2. Find a sequence (a_n) which is non-negative and *strictly* decreasing but where $\sum(-1)^{n+1}a_n$ is divergent and a sequence (b_n) which is non-negative and null but where $\sum(-1)^{n+1}b_n$ is divergent. In both cases, give reasons.

Problem 3. Using the Alternating Series Test where appropriate, show that each of the following series is convergent.

a) $\sum \frac{(-1)^{n+1}n^2}{n^3+1}$ *b)* $\sum \frac{2|\cos\frac{n\pi}{2}|+(-1)^n n}{\sqrt{(n+1)^3}}$ *c)* $\sum \frac{1}{n}\sin\frac{n\pi}{2}$

Problem 4. Is the series $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\log n}$ absolutely convergent? Convergent?

Problem 5. Is it true: "A series is convergent if and only if it is absolutely convergent"? Explain.

Problem 6. Determine for which values of x the following series converge and diverge. [Make sure you don't neglect those values for which the Ratio Test doesn't apply.]

a) $\sum \frac{x^n}{n!}$ *b)* $\sum \frac{n}{x^n}$ *c)* $\sum \frac{(4x)^{3n}}{\sqrt{n+1}}$ *d)* $\sum(-nx)^n$

Problem 7. Prove that if a non-negative sequence (a_n) tends to a and $a > 0$, then $\sqrt{a_n} \to \sqrt{a}$. Prove this, by first showing that

$$\sqrt{a_n} - \sqrt{a} = \frac{a_n - a}{\sqrt{a_n} + \sqrt{a}}.$$