Assignment 3

Due Monday 25 October 15:00 (in supervisor pigeon hole)

1. Prove that each of the following sequences tends to infinity
   (a) \( a_n = n + 10 \)
   (b) \( a_n = \sqrt{n} + \sin n \)
   (c) \( a_n = 2\sqrt{n} \)

2. Prove that none of the following sequences tends to infinity
   (a) \( a_n = 40 - \frac{1}{n} \)
   (b) \( b_n = \cos(n^2 + 7) \)
   (c) \( c_n = 2^{\sin(n\pi)} \)

3. Prove that a sequence which is bounded above cannot tend to infinity.

4. A sequence is known to be increasing.
   (a) Might it have an upper bound?
   (b) Might it have a lower bound?
   (c) Must it have an upper bound?
   (d) Must it have a lower bound?

5. Prove
   (i) Sum Rule
   (ii) Product Rule
   (iii) Quotient Rule
   for sequences.

6. Find the limit of the sequences defined below
   (a) \( \frac{7n^2 + 8}{3n^2 - 4n} \)
   (b) \( \frac{2^n + 1}{2^n - 1} \)
   (c) \( \frac{(\sqrt{n+3})(\sqrt{n-2})}{4\sqrt{n} - 5n} \)
   (d) \( \frac{1 + 2 + \ldots + n}{n^2 + n} \)