Problem 1. Let \( s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \). Use \( |s - s_n| \leq \frac{1}{n} \) to find a value of \( N \) so that \[ \left| \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n} - s \right| \leq 10^{-6}. \]

Problem 2. Find (i) a sequence \((a_n)\) which is non-negative and strictly decreasing but where \(\sum(-1)^{n+1}a_n\) is divergent; (ii) a sequence \((b_n)\) which is non-negative and null but where \(\sum(-1)^{n+1}b_n\) is divergent. In both cases, give reasons.

Problem 3. Using the Alternating Series Test where appropriate, show that each of the following series is convergent:

(a) \( \sum (-1)^{n+1}n^2 \); (b) \( \sum \frac{2|\cos \frac{n\pi}{2}| + (-1)^n n}{(n + 1)^{3/2}} \); (c) \( \sum \frac{1}{n} \sin \frac{n\pi}{2} \).

Problem 4. Is the series \(\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\log n}\) absolutely convergent? Convergent?

Problem 5. Is it true: “A series is convergent if and only if it is absolutely convergent”? Explain.

Problem 6. Determine for which values of \(x\) the following series converge and diverge. [Make sure you do not neglect those values for which the Ratio Test doesn’t apply.]

(a) \( \sum \frac{x^n}{n!} \); (b) \( \sum \frac{n}{x^n} \); (c) \( \sum \frac{(4x)^{3n}}{\sqrt{n + 1}} \); (d) \( \sum (-nx)^n \).

Problem 7. Prove that if \((a_n)\) is a non-negative sequence that tends to \(a\), with \(a > 0\), then \(\sqrt{a_n} \to \sqrt{a} \). Prove this by first showing that

\[ \sqrt{a_n} - \sqrt{a} = \frac{a_n - a}{\sqrt{a_n} + \sqrt{a}}. \]