Assignment 9

Due Monday 6 December 15:00 (in supervisor's pigeon hole)

Problem 1. Let $s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. Use $|s - s_n| \leq \frac{1}{n}$ to find a value of N so that

$$\Big|\sum_{n=1}^{N} \frac{(-1)^{n+1}}{n} - s\Big| \leq 10^{-6}.$$

Problem 2. Find (i) a sequence (a_n) which is non-negative and *strictly* decreasing but where $\sum(-1)^{n+1}a_n$ is divergent; (ii) a sequence (b_n) which is non-negative and null but where $\sum(-1)^{n+1}b_n$ is divergent. In both cases, give reasons.

Problem 3. Using the Alternating Series Test where appropriate, show that each of the following series is convergent:

$$\text{(a)} \quad \sum \frac{(-1)^{n+1}n^2}{n^3+1}; \quad \text{(b)} \quad \sum \frac{2|\cos\frac{n\pi}{2}| + (-1)^n n}{(n+1)^{3/2}}; \quad \text{(c)} \quad \sum \frac{1}{n} \sin\frac{n\pi}{2}.$$

Problem 4. Is the series $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\log n}$ absolutely convergent? Convergent?

Problem 5. Is it true: "A series is convergent if and only if it is absolutely convergent"? Explain.

Problem 6. Determine for which values of x the following series converge and diverge. [Make sure you do not neglect those values for which the Ratio Test doesn't apply.]

$$\text{(a)} \quad \sum \frac{x^n}{n!}; \quad \text{(b)} \quad \sum \frac{n}{x^n}; \quad \text{(c)} \quad \sum \frac{(4x)^{3n}}{\sqrt{n+1}}; \quad \text{(d)} \quad \sum (-nx)^n.$$

Problem 7. Prove that if (a_n) is a non-negative sequence that tends to a, with $a > 0$, then $\sqrt{a_n} \to \sqrt{a}$. Prove this by first showing that

$$\sqrt{a_n} - \sqrt{a} = \frac{a_n - a}{\sqrt{a_n} + \sqrt{a}}.$$