

**Assignment 1**

**Due Monday 10 October 15:00** (in supervisor pigeon hole)

1. Is the following statement true for all  $x$  and  $y$ : “If  $x < y$  then  $x^2 < y^2$ ”?  
What about this statement: “If  $x^2 < y^2$  then  $x < y$ ”?

2. (a) Use mathematical induction to prove that if both  $x$  and  $y$  are positive then  $x < y \Rightarrow x^n < y^n$ .

(b) Now try to prove the *converse*, that if both  $x$  and  $y$  are positive then  $x^n < y^n \Rightarrow x < y$ .

3. Rewrite each of the following expressions without absolute value signs, treating various cases separately where necessary.

$$(a) a - |(a - |a|)|. \quad (b) |(|x| - 2)|.$$

4. Solve the following inequalities:

$$(a) |x - 1| + |x - 2| \geq 5; \quad (b) |x - 1| \cdot |x + 1| > 0.$$

5. Prove the Triangle Inequality. [Hint: Square both sides.]

6. (1) Show, for positive  $a$  and  $b$ , that  $\frac{a+b}{2} - \sqrt{ab} = \frac{(\sqrt{a}-\sqrt{b})^2}{2}$ .

(2) Show that the arithmetic mean of two numbers is always greater than or equal to the geometric mean. When can they be equal?