Analysis I Daniel Ueltschi

Assignment 3

Due Monday 24 October 15:00 (in supervisor pigeon hole)

1. Prove that each of the following sequences tends to infinity
(a) $a_n = n + 10$
(b) $a_n = \sqrt{n} + \sin n$
(c) $a_n = 2^{\sqrt{n}}$

2. Prove that none of the following sequences tends to infinity
(a) $a_n = 40 - \frac{1}{n}$
(b) $b_n = \cos(n^2 + 7)$
(c) $c_n = 2^{\sin(n\pi)}$

3. Prove that a sequence which is bounded above cannot tend to infinity.

4. A sequence is known to be increasing.
(a) Might it have an upper bound?
(b) Might it have a lower bound?
(c) Must it have an upper bound?
(d) Must it have a lower bound?

5. Suppose that $a_n \to a$ and $b_n \to b$. Prove
(i) The Sum Rule: $a_n + b_n \to a + b$.
(ii) The Product Rule: $a_n \cdot b_n \to a \cdot b$
(iii) The Quotient Rule: If $b_n \neq 0$ and $b \neq 0$, $\frac{a_n}{b_n} \to \frac{a}{b}$.

6. Find the limit of the sequences defined below
(a) $\frac{7n^2+8}{4n^2-3n}$
(b) $\frac{2^n+1}{2^n-1}$
(c) $\frac{(\sqrt{n}+3)(\sqrt{n}-2)}{4\sqrt{n}-5n}$
(d) $\frac{1+2+..+n}{n^2}$