

**Assignment 7**

**Due Monday 21 November 15:00** (in supervisor pigeon hole)

**Problem 1.** Consider the series  $1 + 2 + 3 + \dots = \sum_{k=1}^{\infty} k$ . Give a precise explanation of either (i) or (ii).

(i)  $1 + 2 + 3 + \dots = \infty$ .

(ii)  $1 + 2 + 3 + \dots = -\frac{1}{12}$ .

**Problem 2.** Find the sum of the series

(a)  $\sum_{n=1}^{\infty} \frac{1}{10^n}$ .

(b)  $\sum_{n=1}^{\infty} \frac{9}{10^n}$ .

(c)  $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ . Hint: the series  $\sum \frac{1}{2n-1}$  and  $\sum \frac{1}{2n+1}$  may help.

**Problem 3.** Find  $N$  such that

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{N} > 10$$

Give reasons.

**Problem 4.** Prove the Sum Rule for series.

**Problem 5.** Prove the Shift Rule for series.

**Problem 6.** Give an example of a divergent series  $\sum_{n=1}^{\infty} a_n$  for which  $(a_n) \rightarrow 0$ .

**Problem 7.** Prove that if  $\sum_{n=1}^{\infty} a_n$  converges, then  $(a_n) \rightarrow 0$ .

**Problem 8.** Use the Comparison Test to decide whether the following series are convergent or divergent.

(a)  $\sum_{n=1}^{\infty} \frac{1}{n^n}$ .

(b)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ .

(c)  $\sum_{n=1}^{\infty} \frac{n+\sin n}{n^2+\cos n}$ .

(d)  $d. \sum_{n=1}^{\infty} \frac{n \sin n}{4^n}$ .