

### Assignment 9

**Due Monday 5 December 15:00** (in supervisor's pigeon hole)

**Problem 1.** Let  $s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ . Use  $|s - s_n| \leq \frac{1}{n}$  to find a value of  $N$  so that

$$\left| \sum_{n=1}^N \frac{(-1)^{n+1}}{n} - s \right| \leq 10^{-6}.$$

**Problem 2.** Find (i) a sequence  $(a_n)$  which is non-negative and *strictly* decreasing but where  $\sum (-1)^{n+1} a_n$  is divergent; (ii) a sequence  $(b_n)$  which is non-negative and null but where  $\sum (-1)^{n+1} b_n$  is divergent. In both cases, give reasons.

**Problem 3.** Using the Alternating Series Test where appropriate, show that each of the following series is convergent:

$$(a) \sum \frac{(-1)^{n+1} n^2}{n^3 + 1}; \quad (b) \sum \frac{2|\cos \frac{n\pi}{2}| + (-1)^n n}{(n+1)^{3/2}}; \quad (c) \sum \frac{1}{n} \sin \frac{n\pi}{2}.$$

**Problem 4.** Is the series  $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\log n}$  absolutely convergent? Convergent?

**Problem 5.** Is it true: "A series is convergent if and only if it is absolutely convergent"? Explain.

**Problem 6.** Determine for which values of  $x$  the following series converge and diverge. [Make sure you do not neglect those values for which the Ratio Test doesn't apply.]

$$(a) \sum \frac{x^n}{n!}; \quad (b) \sum \frac{n}{x^n}; \quad (c) \sum \frac{(4x)^{3n}}{\sqrt{n+1}}; \quad (d) \sum (-nx)^n.$$

**Problem 7.** Prove that if  $(a_n)$  is a non-negative sequence that tends to  $a$ , with  $a > 0$ , then  $\sqrt{a_n} \rightarrow \sqrt{a}$ . Prove this by first showing that

$$\sqrt{a_n} - \sqrt{a} = \frac{a_n - a}{\sqrt{a_n} + \sqrt{a}}.$$