1. Show that \( \int_0^\infty \frac{\sin x}{x} \, dx = \frac{\pi}{2} \).

Hint: Use \( \int_0^\infty \frac{\sin x}{x} \, dx = \lim_{n \to \infty} \int_0^1 \frac{\sin(nx)}{x} \, dx \). This is almost the integral of the Dirichlet kernel, and we know that \( \int_0^1 D_n(x) \, dx = 1 \). Use Riemann-Lebesgue to show that the difference goes to 0.

2. Properties of the Fourier transform. Suppose that \( f \in L^1(\mathbb{R}) \). Prove the following facts:

   (a) **Riemann-Lebesgue lemma.**
   
   \[ \lim_{|k| \to \infty} \hat{f}(k) = 0. \]

   (b) Convolutions: \( \hat{f} \ast g(k) = \hat{f}(k) \hat{g}(k) \). 

   (c) If \( f(x) \) and \( xf(x) \) are functions in \( L^1(\mathbb{R}^d) \), show that \( \hat{f} \) is differentiable, and that
   
   \[ \frac{d}{dk} \hat{f}(k) = -2\pi i x \hat{f}(k). \]

   (d) If \( f \) and \( \frac{d}{dx} f \) are functions in \( L^1(\mathbb{R}^d) \), show that
   
   \[ \frac{d}{dx} (k) = 2\pi i x \hat{f}(k). \]

3. Let \( d = 1 \). Show that the Fourier transform of \( \frac{1}{x + \mu} \) is \( \frac{\pi}{\mu} e^{-2\pi \mu |k|} \) (use contour methods).

4. Let \( d = 1 \). Show that the Fourier transform of \( e^{-2\pi \mu |x|} \) is \( \frac{\mu}{\pi} \frac{1}{k^2 + \mu^2} \).

5. Show that if \( f \in L^1(\mathbb{R}^3) \) is invariant under rotations, then \( \hat{f} \) is also invariant under rotations.

6. Let \( d = 3 \). Show that the Fourier transform of \( \frac{1}{|x|} e^{-2\pi \mu |x|} \) is \( \frac{1}{\pi} \frac{1}{k^2 + \mu^2} \).