QUANTUM MECHANICS: BASIC PRINCIPLES

& PROBABILISTIC METHODS (MA4A7)

Tuesday 1-2 in MS.03, Thursday 9-10 in B3.08, Friday 2-3 in B3.03.

Course web page: http://www.maths.cam.ac.uk/teaching/2015-MA4A7.html

Description: Brief introduction for mathematicians.

Topics: Wave function in Hilbert space, Schrödinger equation, uncertainty principle, harmonic oscillator, hydrogen atom.

Main mathematical notions: spectral theorem for unbounded operators & Feynman-Kac formula.


(See the excellent introduction in Messiah’s book.)

Until around 1900, Classical Theory reigned supreme. The physical world consisted of matter ( corpuscular ) and radiation (wave-like behaviour). Let $\mathcal{N}$ denote the “state space” of the system, that is, the state of the physical system at any time is encoded in an element $\omega \in \mathcal{N}$. The evolution is described by a function $\psi: [0, T] \rightarrow \mathcal{N}$ that satisfies a first order differential equation. For instance:

- Classical mechanics and Hamilton equations: $x, p \in \mathbb{R}^n$, $L = \mathbb{R}^n$.

Hamiltonian function $H: \mathcal{N} \rightarrow \mathbb{R}$, and

$$\frac{dx}{dt} = \frac{\partial H}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial H}{\partial x}.$$  

(This is equivalent to Newton equations.)

- Electromagnetism and Maxwell equations:

$$\frac{\partial E}{\partial t} = -\nabla \times \mathcal{B}, \quad \frac{\partial B}{\partial t} = \nabla \times \mathcal{E} - \mu_0 \mathcal{J},$$  

( with $\nabla \times \mathcal{E} = \varepsilon_0 \rho$, $\nabla \cdot \mathcal{B} = 0$, $\frac{\partial \mathcal{B}}{\partial t} = -\nabla \times \mathcal{E}$).

It was believed that electromagnetic waves take place in some medium, called “ether”, which might have been a form of matter. It looked like a classical theory of matter could describe the whole physical world.
1887: Michelson & Morley design a careful experiment in order to measure the velocity of Earth through ether. No velocity! The war on ether finally, as the prop of Earth's motion, was not supported by experiments. The concept of ether was totally abandoned in 1905 with Einstein's article on special relativity.

But this did not signal the end of Classical Theory.

At about the same time, the existence of atoms was getting confirmed. Thomson had detected the electron in 1897. Einstein and Smuluchowski had explained the Brownian motion using the atomic hypothesis, and had found estimates of Avogadro's number (1905).

1896: Discovery of radioactivity. Important step conceptually, and even more important experimentally.

1911: Rutherford studies the scattering of α-particles (i.e., He++) by atoms, thus gaining much more information on the properties of the atom. They consist of a nucleus and of electrons that gravitate around.

It should be remarked that the Coulomb force \( F = \frac{q_1 q_2}{4 \pi \varepsilon_0 r^2} \) and the Lorentz force \( F = q(v \times B) \) for charged particles were well understood.

1900: Max Planck's theory of blackbody radiation. He postulated that the exchange of energy between matter and radiation is quantized:

\[
E = h \nu
\]

Planck constant, \( h = \frac{4 \pi}{\lambda_{\text{pl}}} = 6.05 \times 10^{-34} \text{ Js} \)

For most physicists, Planck's theory was a "lucky mathematical accident." But it got further confirmations, notably by Einstein's explanation of the photoelectric effect in 1905. (Einstein got his only Nobel prize for this.)

1924: Compton effect, about the scattering of photons by electrons. It is observed that

\[
\Delta \lambda = \frac{h}{m_c \lambda_{\text{photon}}} \quad \text{or} \quad \lambda_{\text{scattered}} = \lambda_{\text{incident}} - \Delta \lambda
\]

Charge of wavelength of photon does not depend on \( \lambda \).

Compton and Debye showed that this formula can be explained if photons make a single elastic collision with an electron. It confirms Planck's low and the corpuscular nature of light.

On the other hand, experiments about the diffraction and interference of light show that its behavior is wave-like. A purely corpuscular theory of light does not work.