QUANTUM MECHANICS: BASIC PRINCIPLES & PROBABILISTIC METHODS (MA4A7)

Tuesday 1-2 in MS.03. Thursday 9-10 in B3.03. Friday 2-3 in B3.03.

Course web page: http://www.ueltschi.org/teaching/2013-MA4A7.html

<u>Description</u>: Brief introduction for mathematicians.

Topics: Wave functions in Hilbert spaces, Schrödinger equation, uncertainty principle, harmonic oscillator, hydrogen atom.

Main mathematical notions: spectral theorem for unbounded operators & Feynman-Kac formula.

1. Origins of quantum mechanics

(See the excellent introduction in Messiah's book.)

Until around 1900, Classical Theory reigned supreme. The physical world consists of <u>matter</u> (corpuscular) and <u>radiation</u> (wave-like behaviour). Let Ω denote the "state space" of the system, that is, the state of the physical system at any time is encoded in an element $\omega \in \Omega$. The evolution is described by a function $\omega_t : [0,T] \to \Omega$ that satisfies a first order differential eq. in t. For instance:

- Classical mechanics and Hamilton equations: $x, p \in \mathbb{R}^n$, $\Omega = \mathbb{R}^{2n}$, Hamilton function $H : \Omega \to \mathbb{R}$, and
$$\frac{d}{dt} x = \frac{\partial H}{\partial p}, \qquad \frac{d}{dt} p = -\frac{\partial H}{\partial x}.$$
(This is equivalent to Newton equations.)
- Electromagnetism and Maxwell equations:
$$\frac{d}{dt}\vec{B} = -\nabla \times \vec{E}, \qquad \mu_0 \varepsilon_0 \frac{d}{dt}\vec{E} = \nabla \times \vec{B} - \mu_0 \vec{J}$$
(with $\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0}\rho$, $\nabla \cdot \vec{B} = 0$, $\frac{d}{dt}\rho = -\nabla \cdot \vec{J}$).

It was believed that electromagnetic waves take place in some medium, called "ether", which might have been a form of matter. It looked like a classical theory of matter could describe the whole physical world.

1887: Michelson & Morley design a careful experiment in order to measure the ~~s~~ velocity of Earth through ether: no velocity! This was an upsetting result, as the prospect of Earth carrying ether with it did not seem appealing. The concept of ether was totally abandoned in 1905 with Einstein's article on special relativity.

But this did not signal the end of Classical Theory.

At about the same time, the existence of atoms was getting confirmed. Thomson had detected the electron in 1897. Einstein and Smoluchowski had explained the Brownian motion using the atomic hypothesis, and had found estimates of Avogadro's number (1905).

1896: discovery of radioactivity. Important step conceptually, and even more important experimentally.

1911: Rutherford studies the scattering of α-particles (i.e. He^{++}) by atoms, thus gaining much more information on the properties of the atoms. They consist of a nucleus and of electrons that gravitate around.

It should be remarked that the Coulomb force ($\vec{F} = q\vec{E}$) and the Lorentz force ($\vec{F} = q\vec{v} \times \vec{B}$) for charged particles were well understood.

1900: Max Planck's theory of black-body radiation. He postulates that the exchange of energy between matter and radiation is <u>quantised</u>:

$$E_\nu = h\,\nu$$

frequency of radiation

Planck constant, $\hbar = \frac{h}{2\pi} = 1.05 \cdot 10^{-34}\ \mathrm{J \cdot s}$

For most physicists, Planck's theory was a "lucky mathematical artifice". But it got further confirmations, notably by Einstein's explanation of the photoelectric effect in 1905 (Einstein got his only Nobel prize for this).

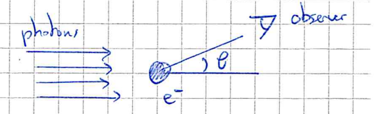
1924: Compton effect, about the scattering of photons by electrons. It is observed that

$$\Delta\lambda = \frac{2h}{mc}\sin^2\frac{\theta}{2}$$

change of wavelength of photon

again Planck's constant!

does not depend on λ



Compton and Debye showed that this formula can be explained if photons make a single elastic collision with an electron. It confirms Planck's law and the corpuscular nature of light.

On the other hand, experiments about the diffraction and interference of light show that its evolution is wave-like. A purely corpuscular theory of light does not work.