## Assignment 3

**Due Monday 30 October 15:00**. Two problems will be selected for marking among those with boxed numbers. There are **4 points** for each of the two problems, and **2 points** for presentation. Do not forget to write **your name** and **your group** (**1**: Tuesday 12-1/Atkinson; **2**: Tuesday 2-3/Vasdekis; **3**: Thursday 1-2/Archer; **4**: Friday 12-1/Bowditch).

**1.** Find the derivatives of the following functions:

1. $F(x) = \int_1^x \log t \,\mathrm{d}t, x > 1$;
2. $G(x) = \int_x^0 \sqrt{1+t^4}\mathrm{d}t, x \in \mathbb{R}$;
3. $H(x) = \int_x^{x^2} \mathrm{e}^{-t^2}\mathrm{d}t, x \in \mathbb{R}$;
4. $I(x) = \int_{\frac{1}{x}}^{\sqrt{x}} \cos(t^2)\mathrm{d}t, x > 0$.

(Hint: Use the chain rule and the fundamental theorem of calculus.)

**2.** Find the following integrals.

1. $\int_0^1 \log(1+x)\,\mathrm{d}x$;
2. $\int_{-2}^{-1} \frac{1}{x^3}\,\mathrm{d}x$;
3. $\int_{-a}^{a} \mathrm{e}^t\,\mathrm{d}t$ for $a \in \mathbb{R}$;
4. $\int_0^a t\,\cos(t^2)\,\mathrm{d}t$ for $a > 0$.

**3.** Evaluate the following improper integrals if they are convergent (or establish their divergence).

1. $\int_0^1 \log x\,\mathrm{d}x$;
2. $\int_0^1 \frac{1}{\sin x}\,\mathrm{d}x$;
3. $\int_0^\infty \sin t\,\mathrm{d}t$;
4. $\int_0^\infty \mathrm{e}^{\cos t}\sin t\,\mathrm{d}t$.

**4.** Prove the Cauchy-Schwarz inequality for regulated functions. Namely, if $f, g \in R[a,b]$, prove that

$$\Big(\int_a^b f(x)g(x)\mathrm{d}x\Big)^2 \leq \int_a^b f(x)^2\mathrm{d}x \int_a^b g(x)^2\mathrm{d}x.$$

(Hint: You can try a proof similar to that of Problem 4 in Assignment 1.)

**5.** Show that the set of discontinuities of $f \in R[a,b]$ is countable. (Hint: Check that the set of jumps of size $\frac{1}{n}$ is finite.)